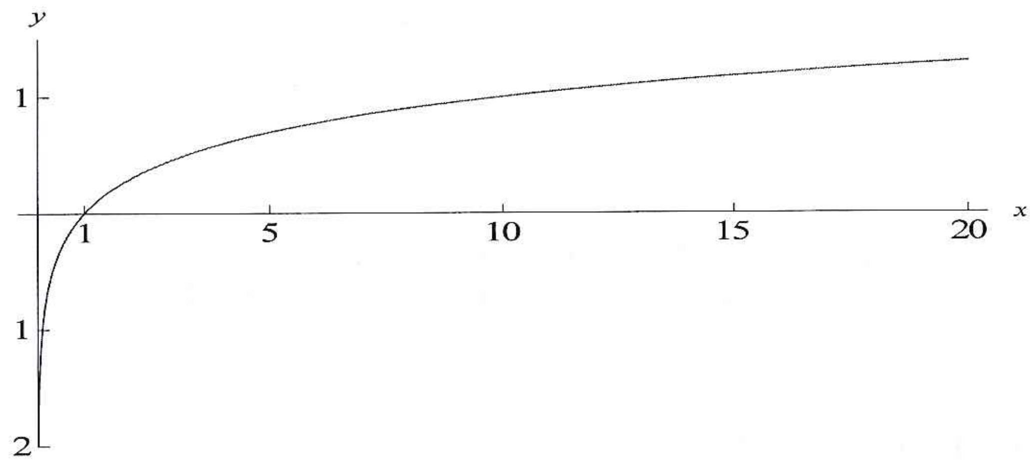
Quiz
Precalc Day 56/57

Sec. 5.3 The Logarithmic Function

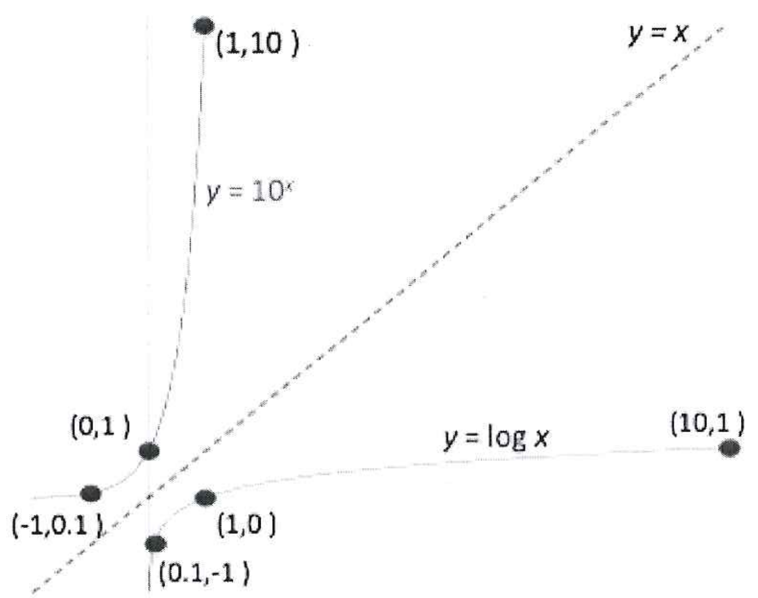
Common Log Function – f(x) = log x if and only if x = 10^y



The **domain** of log x is all positive numbers. Its **range** is all real numbers. The log function grows very rapidly for $0 < x < 1$ and very slowly for $x > 1$. It has a vertical asymptote at $x = 0$ and never touches the y-axis.

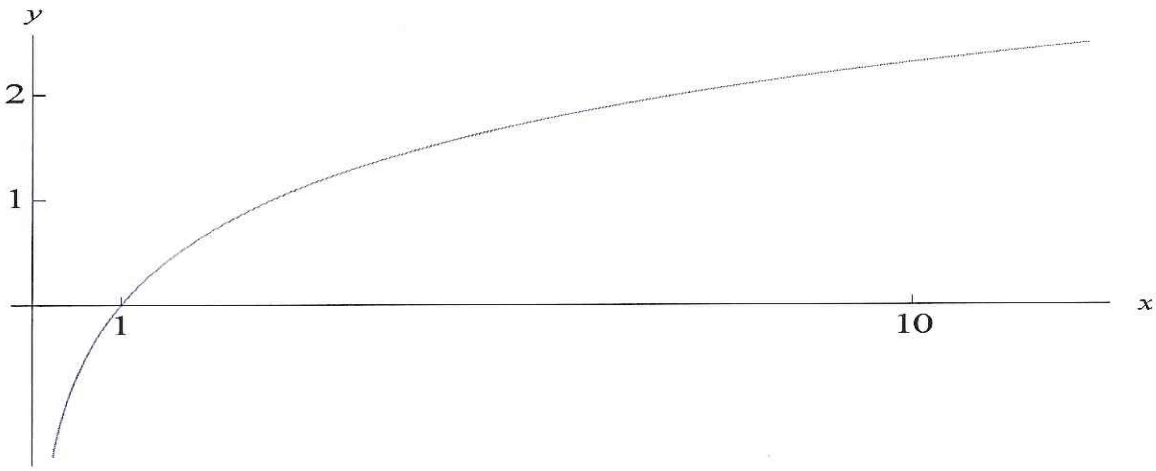
The Graphs of Inverse Functions $y = \log x$ and $y = 10^x$

Exponential function	
x	$y = 10^x$
-2	0.01
-1	0.1
0	1
1	10
2	100
3	1000



Log function	
x	$y = \log x$
0.01	-2
0.1	-1
1	0
10	1
100	2
1000	3

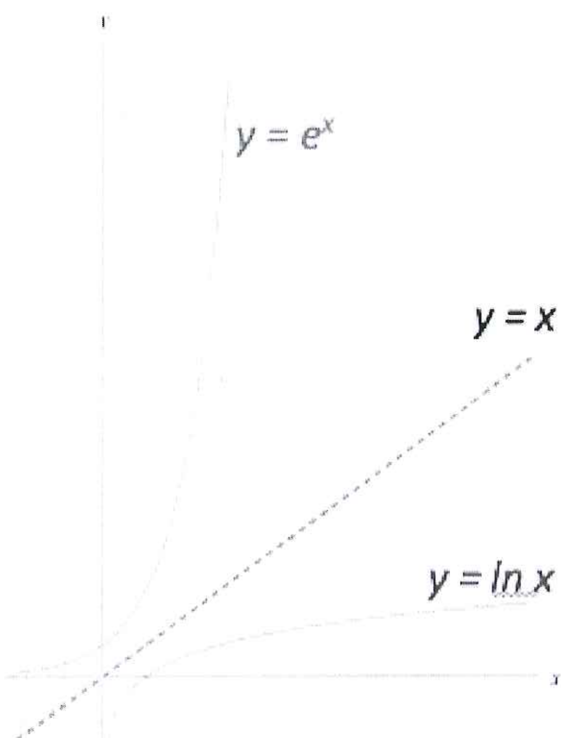
Natural Log Function: y = ln x if and only if x = e^y^



Like the common log, the natural log is only defined for $x > 0$ and has a vertical asymptote at $x = 0$. The graph is slowly increasing and concave down. It also passes through (1, 0).

The Natural Logarithm and Its Inverse

The functions $y = \ln x$ and $y = e^x$ are inverses of one another. Notice how they are mirror images of one another through the line $y = x$.



Properties of Log Functions f(x) = $\log_a x$:

1. The domain is the set of positive real numbers and the range is all real numbers.
2. The x-intercept of the graph is 1. There is no y-intercept.
3. The y-axis is a vertical asymptote of the graph.
4. A log function is decreasing if 0 < a < 1 and increasing if a > 1.
5. The graph of f contains the points (1, 0), (a, 1), and (1/a, -1).
6. The graph is continuous with no gaps or corners.

$y = \log_7 x$

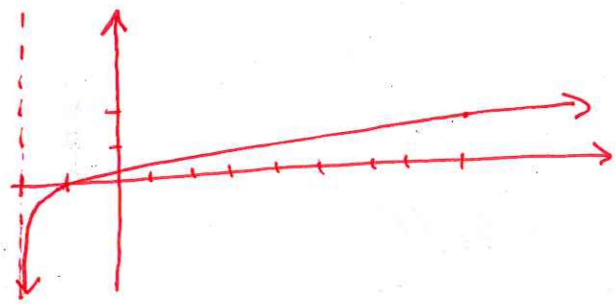
$x = 7^y$

$1 = 7^0 \quad (1, 0)$

$7 = 7^1 \quad (7, 1)$

$\frac{1}{7} = 7^{-1} \quad (\frac{1}{7}, -1)$

Ex. Graph f(x) = log(x + 2) by starting with the graph of f(x) = log x. Determine the domain, the range, the vertical asymptote, and the x-intercept.



domain: $x > 2$

range: all reals

VA: $x = -2$

x-int: $(-1, 0)$

Asymptotes and Limit Notation

Let $y = f(x)$ be a function and let a be a finite number.

- The graph of f has a **horizontal asymptote** of $y = a$ if

$$\lim_{x \to \infty} f(x) = a \quad \text{or} \quad \lim_{x \to -\infty} f(x) = a \quad \text{or both.}$$

- The graph of f has a **vertical asymptote** of $x = a$ if

$$\lim_{x \to a^+} f(x) = \infty \text{ or } \lim_{x \to a^+} f(x) = -\infty \text{ or } \lim_{x \to a^-} f(x) = \infty \text{ or } \lim_{x \to a^-} f(x) = -\infty.$$

Ex. Find: a) $\lim\limits_{x \to 0^+} \log x$ b) $\lim\limits_{x \to -2^+} \log(x+2)$ c) $\lim\limits_{x \to \infty} \ln x$ d) $\lim\limits_{x \to -\infty} 10^x$

a) $-\infty$ b) $-\infty$ c) ∞ d) 0

VA: $x = 0$ VA: $x = -2$ HA: $y = 0$

Ex: The sound intensity of a refrigerator motor is 10^{-11} watts/cm^2. A typical school cafeteria has sound intensity of 10^{-8} watts/cm^2. How many orders of magnitude more intense is the sound of the cafeteria?

$\frac{10^{-8}}{10^{-11}} = 10^3 = 1{,}000$ times more intense

Noise Level in Decibels = $10\log\left(\frac{I}{I_0}\right)$ where I represents the sound's intensity and is compared to the intensity of a benchmark sound I_0. The intensity of I_0 is defined as 10^{-16} watts/cm^2, roughly the lowest intensity audible to humans.

Ex: What is the noise level (in dB) of the refrigerator in the previous example?

$= 10\log\left(\frac{10^{-11}}{10^{-16}}\right)$

$= 10\log 10^5$

$= 10 \cdot 5$

$= 50\,dB$

Ex: If a sound doubles in intensity, by how many units does its decibel rating increase?

$10\log\left(\frac{2I}{I_0}\right) - 10\log\left(\frac{I}{I_0}\right)$

$10\left(\log\left(\frac{2I}{I_0}\right) - \log\left(\frac{I}{I_0}\right)\right)$

$10\left(\log\left(\frac{2I}{I_0} \div \frac{I}{I_0}\right)\right)$

$10\log\left(\frac{2I}{I_0} \cdot \frac{I_0}{I}\right)$

$\to 10\log 2$

$\approx 3.010\,dB$

Ex: Loud music can measure 110 dB whereas normal conversation measures 50 dB. How many times more intense is loud music than normal conversation?

$10\log\left(\frac{I_m}{I_0}\right) = 110$

$\log\left(\frac{I_m}{I_0}\right) = 11$

$10^{11} = \frac{I_m}{10^{-16}}$

$10^{11} \cdot 10^{-16} = I_m$

$10^{-5} = I_M$

$10\log\left(\frac{I_c}{I_0}\right) = 50$

$\log\left(\frac{I_c}{I_0}\right) = 5$

$10^5 = \frac{I_c}{10^{-16}}$

$10^5 \cdot 10^{-16} = I_c$

$10^{-11} = I_c$

$\frac{I_m}{I_c} = \frac{10^{-5}}{10^{-11}} = 10^6$

$10^6 = 1{,}000{,}000$ times more intense

HW: pg 203-205, #1-9,11,12,19,20,22-25,40 (#25—Intensity instead of Loudness)